Prof. Rüdiger Frey
Institute for Statistics and Mathmatics
WU Vienna
ruediger.frey@wu.ac.at

Master QFin, CTFI
Final Exam, Fr 23.6. 11.00-12.30

Hints

- This is a closed-book exam.
- Good luck !

1. Ito calculus

a) (2 points) Consider a twice differentiable function F on $\mathbb{R}$ and a Brownian motion W. Use the Ito formula to show that $F(W_t) - \frac{1}{2}\int_0^t F''(W_s)ds$ is a (local) martingale.

b) (Square root of geometric Brownian motion, 2 points). Suppose that $dS_t = \mu S_t dt + \sigma S_t dW_t$ with initial value S_0. Compute the dynamics of $\sqrt{S_t}$ via Ito's formula.

2. Black Scholes model and binary option. Consider in the context of the Black Scholes model with stock price dynamics $dS_t = \mu S_t dt + \sigma S_t dW_t$, initial stock price $S_0 > 0$ and with money market account $B_t = \exp(rt)$ for $r > 0$ a so-called binary option with payoff $h(S_T) = 1_{\{S_T > K\}}$ for some $K > 0$.

a) (2 points) Write down the terminal value problem for the fair price $u(t, S)$ of the option.

b) (2 points) *Derive* the risk-neutral pricing formula for the option via Feynman Kac.

c) (4 points) Use the result of b) to show that the price of the option at time t equals

$e^{-r(T-t)} \cdot N(d_2)$

$$u(t, S) = N(d_2) \text{ where } d_2 = \frac{\ln\left(\frac{S}{K}\right) + (r - \frac{1}{2}\sigma^2)(T-t)}{\sigma\sqrt{T-t}},$$

and where N is the standard normal distribution function.

d (4 points) Construct a delta neutral hedging strategy for the option and compute explicitly the stock position. Discuss qualitatively potential problems in the implementation of the strategy. Hint: consider the value of the option delta for $S \approx K$ and a small time to maturity.

e) (2 points) Which sign would you expect for the Vega of the option. Distinguish the case where S is substantially lower than K and the case where $S > K$. (An intuitive ecomomic argument is enough, a formal computation is not necessary).